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FATIGUE STRESS CALCULATION OF STRAIGHT BEVEL GEARS APPLIED TO A PHOTO VOLTAIC TRACKING SYSTEM

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***Abstract*—** Gear transmissions are frequently used in PV tracking system because of the possibility to realize very large angular movements. The paper presents a new straight bevel gear system for large dimension photo voltaic (PV) platforms. The PV tracking movement's errors depending by straight bevel gearing errors and also by the tracking elements deflections. The design accuracy of straight bevel gear is influencing the load transmission, the size and gear weight and also the system performances. The paper analyses the influence of the geometry of straight bevel gears on the combined geometry factor for the pinion and also the wheel of a straight bevel gear from a tracking system transmission, for static load.

***Keywords*—** Combined geometry factor, fatigue stress, straight bevel gears for tracking system.

I. INTRODUCTION

For collected solar radiation maximizing, mono axis and dual axis tracking systems are used. The dual axis tracking systems advantage, given by the energetic gain, compared with mono axis systems, led to an increased interest to study and innovate this and also to optimize and implement them. The two movements of dual axis tracking system can be realized by linear actuators, rotational actuators with gears, or combining them. The gearing transmissions, even expensive than linear actuators, are preferred for photo voltaic (PV) platforms because of very large angular movements (360 degrees) and have to satisfy certain conditions [1]: to run at a reduced rotational speed, usually smaller than one rotation per minute in order to allow a very precise positioning of the platform; to have reduced running hours (approximately 500 hours over 20 years); to have a reduced overall size and high efficiency.

A new simplified straight bevel gear system, presented in Fig. 1., is used for large dimension PV platform dual axis tracking system and is a simplified version of that presented in [2]–[5]. The two tracking movements for PV platform 7, which are sequential in 5...15 degrees steps, are obtained by engaging or disengaging of the breaks C1 and C2 combined by the electrical motor rotation transmitted to the warm gear 1-2. As a result, the gearbox 6 realize the azimuthal movement and the platform 7 the altitude movement.

The proposed tracking system is of azimuthal type. It allows two independent rotational movements related to the following axes: a vertical axis for the azimuthal rotation and a horizontal axis for the altitudinal rotation. The rotations performed by this tracking system are: the azimuthal movement, around vertical axis I, controlled by the C1 coupling and the altitudinal movement, around the horizontal axis II, controlled by the C2 couplings.

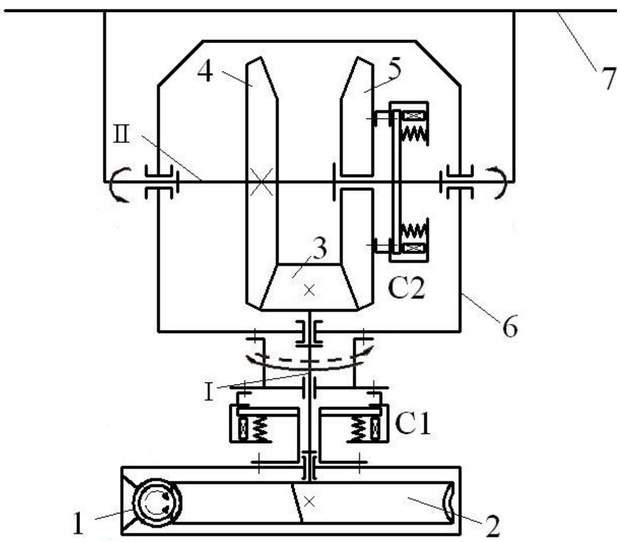


Fig. 1. Conceptual scheme of tracking system

The system correct functioning depending by the following requests [6]: the platform precise positioning relating to minimize the sun rays incident angle errors, the system running life off reduced, an increased energetically efficiency, overall dimension reduced, increased dynamic load as a result of random action of wind. Fulfilling these conditions depends by the correct design of double bevel gear, 3-4 and 3-5, which realize the dual axis movements.

II. THEORETICAL ASPECTS

To reduce the transmission errors, the straight bevel gears need be high precision manufactured. Due to a many complex factor such as the large range of bearing vibration amplitudes, gearing manufacture tolerances, gearing contact deformations, gear bending deflections, shafts bending deflections and torsion deformations, bearing clearances, is difficult to avoid the gearing teeth impact and load concentration [7]. The straight bevel gear teeth contact is given by an elongated ellipse, the gear overall dimensions depending by the teeth contact length. Is absolutely needed that the contact patch to cover the entire teeth length for full load gearing. But, this is impossible because of the teeth manufacturing errors and also gearing assembling errors [8].

The straight bevel design request labored calculus, the needed time increased, prone to errors and repeated calculus and doubling the expenses. So, the straight bevels design quality influencing the gearing overall dimension and weight and also the transmission performances [9]. The straight bevel gear main stresses are contact and bending. Under the external loads, the gears and the platform structure can be damaged by the overloads and also by the material fatigue.

The straight bevel gear geometric parameters influence on the contact stress was analyzed in [10] and static bending stress was analyzed in [4]. For fatigue stress, the large number of loads (because of the wind repeated action) can damage the gear during time, even when these loads produced stresses not exceed the permissible root stress. From all the platform external loads (as wind, rain, snow, seismic, system weight), the most important is the wind action load. The straight bevel gear calculus is based on the calculus methods given by the ISO/TC 60 Gears Technical Committee, in the ISO 10300 international standard.

The angle between the shafts of the bevel gears used in the tracking systems of PV platforms is Σ=90°, so these types of gears are orthogonal bevel gears (Fig. 2. [10], [11]). These types of gears can be manufactured only with non-shifted profiles (x_{hm1}=0 and x_{hm2}=0) or zero shifted profiles (x_{hm2}=-x_{hm1}).

In stress calculus the bevel gears are replaced by equivalent spur cylindrical gears, which is the virtual gear (*Tredgold* approximate) which fulfil the following conditions [4], [11], [12]:

- The pitch circle radius ($d_{v1,2}/2$) of the virtual wheels are equal to the generator lengths of the mean frontal cones of bevel wheels;
- Virtual gear module is equal to the mean module m_m of bevel gear;
- Teeth height of virtual gear is equal to mean height of bevel gear teeth;
- Tangential force from virtual gear is equal to the force from real bevel gear, which is calculated at the pitch diameter level.

The bending stress calculus method B2, presented by ISO 10300-3 [13] and ANSI/AGMA 2003-B97 [14], is based on the bending cantilever theory, modified in order to consider the following elements [13]: the radial component of the normal force F_n creates a compression stress in the dangerous section of tooth root; the fillet of the tooth root is a high stress concentrator for the bending stress; the normal force is distributed between the teeth pairs simultaneously in gearing; the wheel has an irregular movement due to the reduced contact ratio of the straight bevel gear.

According to B2 method, the fatigue bending stress take into consideration the teeth geometry, manufacturing accuracy, the mesh stiffener, bearing and housing, and also the gearing load, and is given by relation [13], [14]

$$\sigma_{F-B2} = \frac{F_{mt} K_A K_\upsilon K_{F\beta} K_{F\alpha}}{b m_m} Y_P \le \sigma_{FP} \quad (1)$$

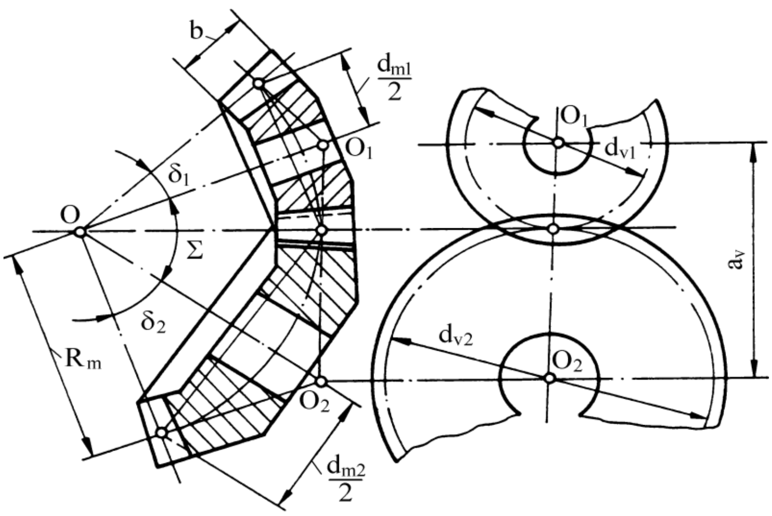


Fig. 2. Virtual gear

where: $\mathbf{F_{mt}} = 2\mathbf{T_1}/\mathbf{d_{m1}}$ is nominal tangential force at reference cone at mid face width of pinion; K_A – application factor; K_v – dynamic factor; $K_{F\beta}$ – face load factor for bending stress; $K_{F\alpha}$ – transverse load factor for bending stress; b – face width; m_m – mean module.

Combined geometry factor Y_P is given by relation

$$\mathbf{Y_P} = \mathbf{Y_A}/\mathbf{Y_J}\left(\mathbf{m_m}/\mathbf{m_e}\right)^2, \tag{2}$$

where: Y_A is bevel gear adjustment factor; Y_J – bevel geometry factor; m_e – outer module (standard for straight bevel gear).

Bevel geometry factor (B2 method) Y_J is given by relation

$$\mathbf{Y_J} = \frac{\mathbf{Y_K}}{\varepsilon_\mathbf{N}\mathbf{Y_i}}\,\frac{2\mathbf{r}_{\mathbf{my}0}}{\mathbf{d_v}}\,\frac{\mathbf{b_{ce}}}{\mathbf{b}}\,\frac{\mathbf{m_m}}{\mathbf{m_e}}, \tag{3}$$

and bevel gear adjustment factor Y_A by relation

$$\mathbf{Y_A} = \frac{\mathbf{Y_f}}{2.3\left(1-\dfrac{\mathbf{s_N}}{3\mathbf{h_N}}\tan\boldsymbol{\alpha}\right)}. \tag{4}$$

Bevel gear factor Y_K depends by tooth thickness at critical section $2s_N$, load height from critical section h_N and tooth fillet radius at the mean section, r_{mf}, and is given by relation

$$\mathbf{Y_K} = \frac{\mathbf{Y}}{\mathbf{Y_f}}, \tag{5}$$

where:

$$\mathbf{Y} = \frac{2}{3}\left[\frac{1}{\dfrac{\mathbf{m_e}}{\mathbf{s_N}}\left(\dfrac{\mathbf{h_N}}{\mathbf{s_N}} - \dfrac{\tan\boldsymbol{\alpha}_\mathbf{h}}{3}\right)}\right] \tag{6}$$

is tooth form factor;

$$\mathbf{Y_f} = \mathbf{L} + \left(\frac{2\mathbf{s_N}}{\mathbf{r_{mf}}}\right)^{\mathbf{M}}\left(\frac{2\mathbf{s_N}}{\mathbf{h_N}}\right)^{\mathbf{O}} \tag{7}$$

is stress concentration and stress correction factor; L, M and O are empirical constants used in stress correction formula.

The tooth fillet radius at the mean section r_{mf} is given by relation

$$\mathbf{r_{mf}} = \frac{\left(\mathbf{h_{fm}} - \boldsymbol{\rho}_{\mathbf{a}0}\right)^2}{\mathbf{d_v}/2 + \mathbf{h_{fm}} - \boldsymbol{\rho}_{\mathbf{a}0}} + \boldsymbol{\rho}_{\mathbf{a}0} \tag{8}$$

where h_{fm} is mean dedendum; ρ_{a0} – cutter edge radius; d_v – reference diameter of virtual cylindrical gear.

Other notations used in the above relations have the following meanings: ε_N – load sharing ratio; Y_i – inertia factor; r_{my0} – mean transverse radius to point of load application; d_v – reference diameter of virtual cylindrical gear; b_{ce} – calculated effective face width; α_h – normal pressure angle at point of load application on the tooth centerline. All of these factors can be obtained following ISO 10300-3 [13] and ANSI-AGMA 2003-B97 [14] prescriptions.

The tooth root stress calculus need to be done separately for pinion and also for wheel. For an increased degree of generality, all the geometrical elements are determined related to mean module m_m.

III. RESULTS AND CONCLUSIONS

To analyze the geometrical parameters influence on the bending fatigue stress the authors was elaborated a program for calculus and trace diagrams. The program main menu, presented in Fig. 3., is divided in three parts: *Inputs*, *Calculus* and *Results*. In Fig. 3, the input choosing are also presented.

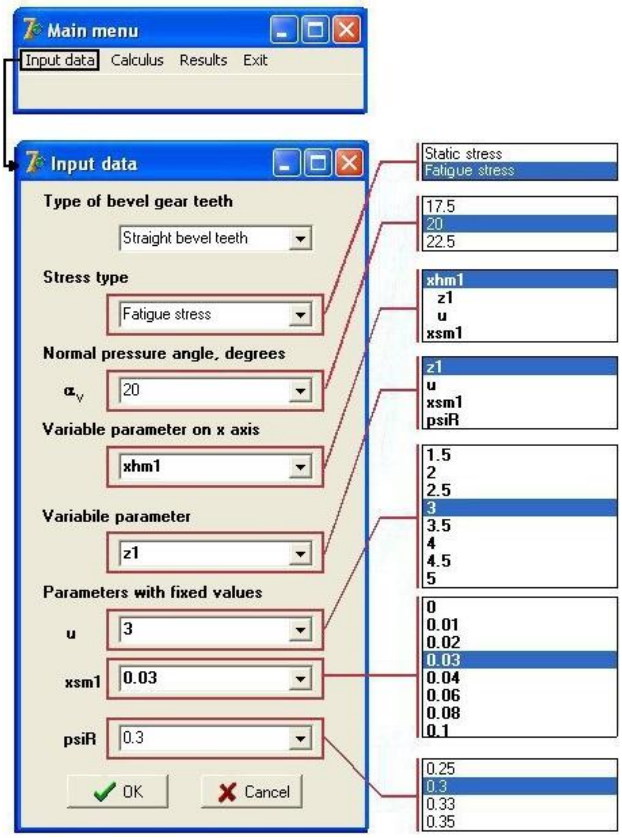


Fig. 3. The main menu and the input

In *Calculus* section, based on the ISO 10300-1 [15], ISO 10300-3 [13] and ANSI/AGMA 2003-B97 [14] relations, the values of $Y_{1,2}$, $Y_{A1,2}$, $Y_{J1,2}$ and $Y_{P1,2}$ factors are determined. The user can choose the output of these values, as table or as diagram, in *Output* section, like shown in Fig. 4.

Because is hard to predict the variation of wind speed, which load finally the tracking system bevel gears, the fatigue calculus becomes very important and required. In this way, the gear damages because of material fatigue can be avoided.

So, it is necessary to analyze geometrical parameters influence on the bending fatigue stress, because these results are real useful for PV tracking designers.

In Fig. 5 are presented the variation of combined geometrical factors Y_{P1}, for pinion and Y_{P2}, for wheel, depending by pinion profile shift coefficient x_{hm1} for $x_{hm2}=-x_{hm1}$, respectively pinion teeth number z_1.

From diagrams, the following conclusions result:

For the pinion, the combined geometrical factor Y_{P1} decrease with profile shift coefficient x_{hm1} and teeth number z_1 increasing;

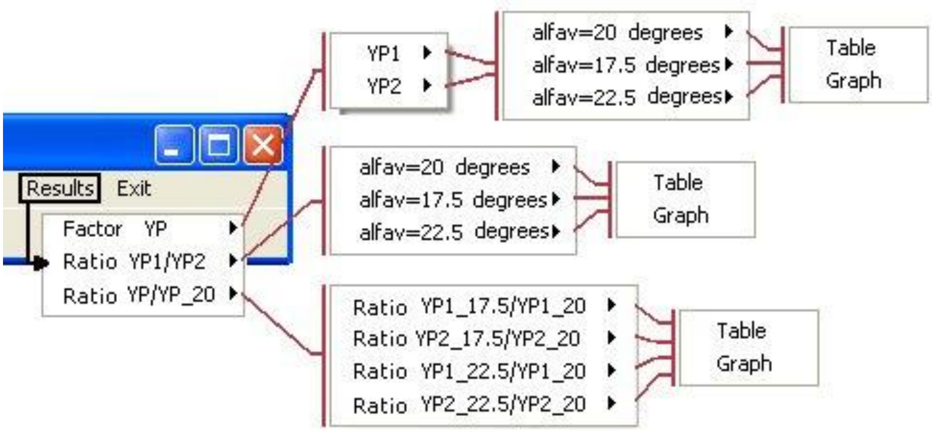
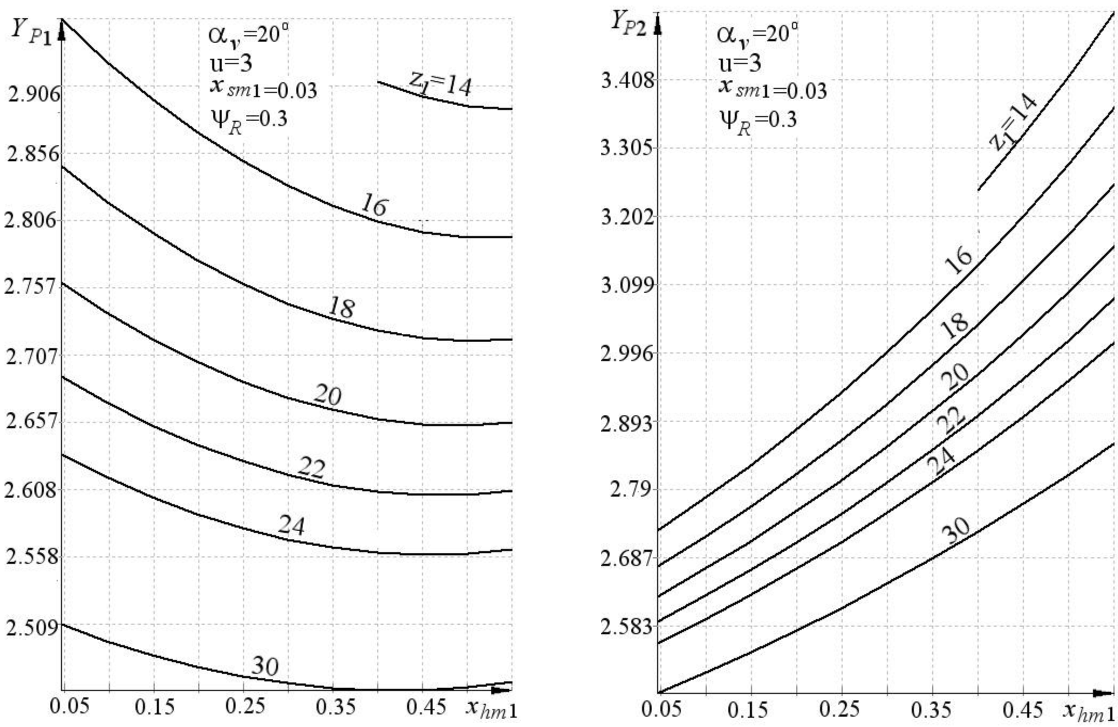


Fig. 4. The program output section



a b

Fig. 5. The Y_{P1} and Y_{P2} factors variation depending by x_{hm1} and z_1

- For the pinion, the combined geometrical factor Y_{P1} decrease with profile shift coefficient x_{hm1} and teeth number z_1 increasing;
- For the wheel, the combined geometrical factor Y_{P2} increase with profile shift coefficient x_{hm1} increasing and also decrease with teeth number z_1 increasing; the increasing of combined factor Y_{P2} is more than the decrease of the combined factor Y_{P1} (as example, for z_1=22 teeth and x_{hm1}=0.4 related to x_{hm1}=0.05, Y_{P1} decrease with 3.2%, and Y_{P2} increase with 19%);
- The Y_{P2} factor increasing is explained by negative value of wheel profile shift coefficient, so the teeth width in dangerous section is decreased related to the pinion dangerous section which increase because x_{hm1}>0.

In Fig. 6 are presented the variation of combined geometrical factors Y_{P1}, for pinion and Y_{P2}, for wheel, depending by pinion profile shift coefficient x_{hm1} for respectively bevel gear ratio u.

From diagrams, the following conclusion result:

- For the pinion, the combined geometrical factor Y_{P1} variation has the same shape as the variation presented in figure 5, the biggest value results for smaller bevel gear ratio;
- For the wheel, the combined geometrical factor Y_{P2} increase with profile shift coefficient x_{hm1} increasing and also decrease with the bevel gear ratio u increasing; the increasing of combined geometric factor Y_{P2} is sensitive more than the decrease of the combined factor Y_{P1}

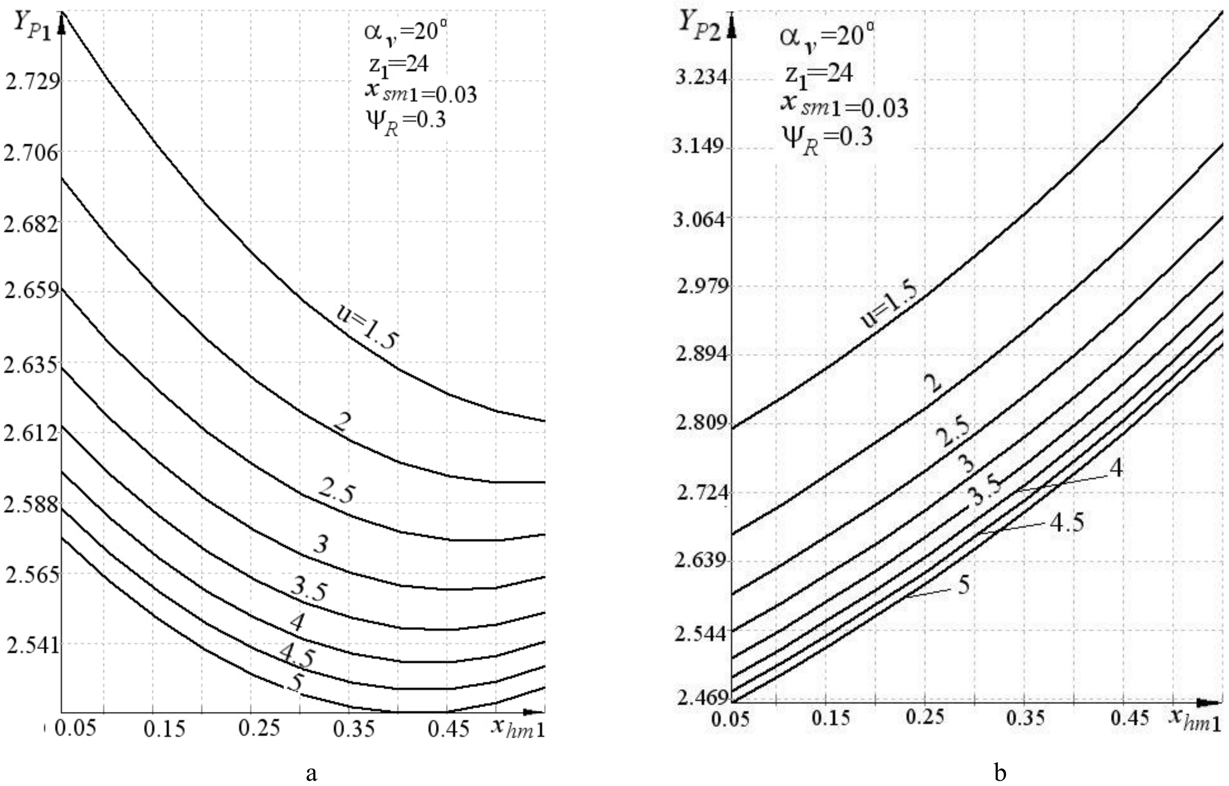


Fig. 6. The Y_{P1} and Y_{P2} factors variation depending by x_{hm1} and u

(as example, for u=4 and x_{hm1}=0.4 related to x_{hm1}=0.05, Y_{P1} decrease with 2.5%, and Y_{P2} increase with 18%).

In Fig. 7. are presented the variation of combined geometrical factors Y_{P1}, for pinion and Y_{P2}, for wheel, depending by pinion profile shift coefficient x_{hm1} respectively by thickness modification coefficient x_{sm1}. From diagrams, the following conclusions result:

- For the pinion, the combined geometrical factor Y_{P1} decrease with thickness modification coefficient x_{sm1} increase, respectively with profile shift coefficient x_{hm1} increasing; the factor increasing is more relevant at big values of x_{hm1} coefficient and smaller values of x_{sm1} coefficient;

For the wheel, the combined geometrical factor Y_{P2} increase with profile shift coefficient x_{hm1} increasing and also decrease with thickness modification coefficient x_{sm1} increase.

IV. CONCLUSION

From previously considerations, the main conclusion result: *a reduced value for fatigue bending stress for straight bevel gear, can be obtained by choosing a bigger pinion teeth number, an increased bevel gear ratio, a bigger value for the thickness modification coefficient and also the profile shift coefficient value around* 0, 4...0,45 *(maximum* 0,5*)*. In these conditions, the number of load cycles is strongly reduced, results an increased permissible stress and an increase of Y_{P2} factor, so the teeth resist to the bending stress.

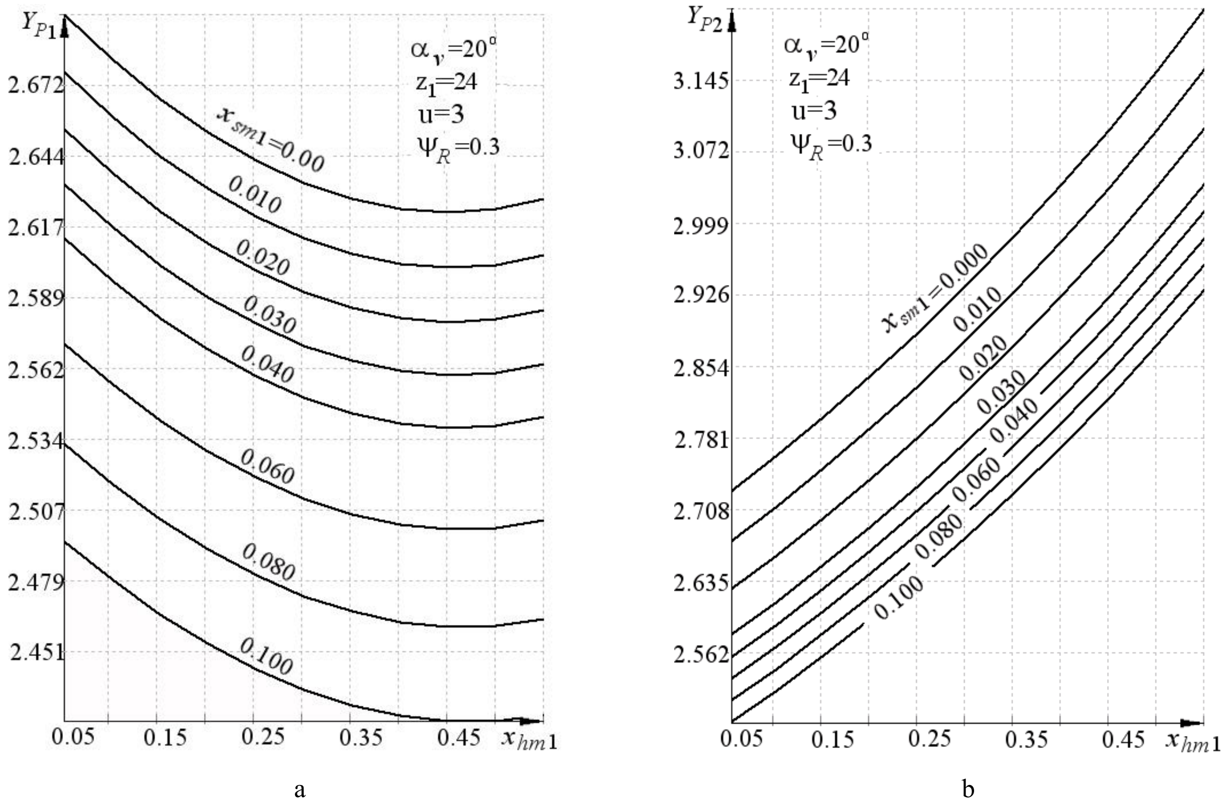


Fig. 7. The Y_{P1} and Y_{P2} factors variation depending by x_{hm1} and x_{sm1}

REFERENCES

[1] B. Butuc, G. Moldovean, C. C. Gavrila, "Computer program for calculus of the straight bevel gears used in the tracking systems of the photovoltaic platforms". Proceedings of the 2nd International Conference ADEMS'09, Cluj-Napoca 2009, pp. 205-210.

[2] B. R. Butuc, R.-G. Velicu, G. Moldovean, , "*Dual axis tracking system with a single actuator*, *Sistem de orientare după două axe cu un singur motor*, (Patent no. RO 126150)", 2009.

[3] B. Butuc, G.. Moldovean, R. Velicu, "Wind and weight induced loads on a gear based azimuthal photovoltaic platform", *Renewable Energy and Power Quality Journal*, No. 9, 2011, Available on: http://www.icrepq.com/rev-papers.htm.

[4] G. Moldovean, B. Butuc, R. Velicu, "Dual axis tracking system with a single motor", *New Trends in Mechanism Science, Analyses and Design, Series Mechanism and Machine Science*, Springer, 2010, pp. 649 – 656.

[5] G. Moldovean, B. Butuc, R. Velicu, "Shafts design of a gear based azimuthal tracked photovoltaic platform", *Environmental Engineering and Management Journal*, Vol.10, No. 9, 2011, pp. 1291-1298.

[6] G. Moldovean, I. Vişa, R. Velicu, B. Butuc, "Static stress calculation of straight bevel gears applied to a PV tracking system", *The First IFToMM Asian Conf. on Mechanism and Machine Science*, 2010, Taipei, China.

[7] D. Yang, H. Cui, X. Tian, Q. Zhang, P. Xu, " Research on tooth modification of spur bevel gear, *The Open Mechanical Engineering Journal*, 5, 2011, pp. 68-77. Available on http://benthamscience.com/open/tomej/articles/V005/68TOMEJ.pdf.

[8] P. Xu, X. Tian, H. Cui, "Contact performance study of modified spur bevel gear considering assembly error", *Advanced Materials Research*, Vol. 421, 2012, pp. 102-106.

[9] X. Zhang, Y. Rong, J.Yu, L. Zhang, L. Cui, "Development of optimization design software for bevel gear based on integer serial number encoding genetic algorithm", *Journal of Software*, Vol.6, Issue 5, 2011, pp. 915-922.

[10] B. R. Butuc, G. Moldovean, R. Velicu, "On the influence of geometry over the contact stress of straight bevel gears", *Machine Design*, Novi Sad 2010, pp. 153-158,.

[11] G. Moldovean, D. Velicu, R. Velicu, A. Jula, E. Chisu, I. Visa, *Cylindrical and Bevel Gears. Calculus and construction*, *Angrenaje cilindrice şi conice. Calcul si constructie*, Brasov, Ed. Lux Libris, 2001.

[12] D. Velicu, G. Moldovean, R. Velicu, *Design of Bevel and Hypoid Gears*, *Proiectarea angrenajelor conice si hipoide*, Brasov, Ed. Universitatii Transilvania din Brasov, 2004.

[13] ISO 10300-3, "*Calculation of Load Capacity of Bevel Gears*, Part 3, *Calculation of Tooth Root Strength*, (*International Standard, First Edition*)", ISO, 2001, Switzerland.

[14] ANSI/AGMA 2003_B97, "*Rating the pitting resistance and bending strength of generated straight bevel, zerol bevel and spiral bevel gear teeth (International Standard)*", American Gear Manufacturers Association, 1997.

[15] ISO 10300-1, "*Calculation of load capacity of bevel gears*, Part 1, *Introduction and General Influence Factors*, (*International Standard, First Edition*)", ISO, 2001, Switzerland.